

**Ricci-Flat and Charged Wormholes in Five Dimensions**H. Lü<sup>†,‡</sup> and Jianwei Mei<sup>†</sup>*<sup>†</sup>George and Cynthia Woods Mitchell Institute for Fundamental Physics and Astronomy,  
Texas A&M University, College Station, TX 77843, USA**<sup>‡</sup>Interdisciplinary Center of Theoretical Studies, USTC, Hefei, Anhui 230026, PRC***ABSTRACT**

We construct stationary Ricci-flat inter-universe Lorentzian wormhole solutions in all  $D \geq 5$  dimensions that connect two flat asymptotic spacetimes. Such a solution can be viewed as the gravity dual of a string tachyon state whose linear momentum is larger than its tension. We focus our analysis on the  $D = 5$  wormholes which are not traversable for the timelike and null geodesics; however, we demonstrate that there exist accelerated timelike trajectories that traverse from one asymptotic region to the other. We further study the minimally-coupled scalar wave equation and demonstrate that the quantum tunnelling between two worlds must occur. We also obtain charged wormholes in five-dimensional supergravities. With appropriate choice of parameters, these wormholes connect  $\text{AdS}_3 \times S^2$  in one asymptotic region to flat Minkowskian spacetime in the other.

# 1 Introduction

Einstein's discovery of general relativity has since inspired people's imagination of taking a "shortcut" in spacetime travel. The first non-trivial wormhole construction is called the Einstein-Rosen bridge that combines a Schwarzschild black hole and a white hole [1], which unfortunately turns out to be rather short lived [2]. However, the existence of wormhole is not inconsistent with general relativity. Most of the studies of this subject have been focused on four dimensions, with either Lorentzian or Euclidean signatures. Typically a wormhole requires a rather unusual energy-momentum tensor to maintain.

With the advent of string theory that demands higher spacetime dimensions, it is natural to examine the possibility of wormholes beyond four dimensions. Euclidean wormholes in string theory have recently been studied [3, 4, 5, 6, 7] as solutions of supergravities. In this paper, we construct stationary Ricci-flat inter-universe Lorentzian wormholes in all  $D \geq 5$  dimensions. The solution is presented in section 2. Using  $D = 5$  as an example, we demonstrate that it is a smooth solution that links two asymptotic flat Minkowskian spacetimes. We find that the solution can be viewed as a gravity dual of a string tachyon state that has a linear momentum larger than its tension. The solution has a BPS limit that leads to a supersymmetric pp-wave, and the two worlds then become disconnected by the pp-wave singularity.

We analyze the geodesic and non-geodesic motions in section 3. The wormholes are not traversable for the timelike or null geodesics. However, we obtain accelerated timelike trajectories that can traverse from one asymptotic region to the other. We demonstrate that the maximum acceleration along the trajectories can be arbitrarily small, and the total impulse required to cross over is finite, and independent on the size of the wormhole.

We then consider the quantum tunnelling of the wormholes in section 4 by examining the minimally-coupled scalar wave equation. We demonstrate that the radial wave equation can be reduced to a one-dimensional quantum mechanical system with a finite potential barrier. This implies that the quantum tunnelling must take place. We obtain the approximate transmission rate using the delta function approximation for the potential. The transmission rate is related to the size of the wormhole, and it vanishes when the wormhole vanishes.

In section 5, we obtain both electrically and magnetically charged wormhole solutions in five-dimensional supergravities. The later one can be viewed as the magnetically charged tachyonic string. Although the solutions become more complicated, the essential feature of the neutral Ricci-flat wormhole remains. With appropriate choice of parameters, we find the charged wormholes that connect  $\text{AdS}_3 \times S^2$  in one end to flat Minkowskian spacetime

in the other.

We conclude the paper in section 6. And finally in the appendix, we present the general wormhole solution in five-dimensional  $U(1)^3$  supergravity that carries both electric and magnetic charges.

## 2 The metric

The Ricci-flat Lorentzian wormhole solution in all dimensions  $D \geq 5$  is given by

$$ds^2 = (r^2 + a^2)d\Omega_{D-3}^2 + \frac{r^2 dr^2}{(r^2 + a^2)\sin^2 h(r)} - \cos(\gamma + f)dt^2 + \cos(\gamma - f)dz^2 + 2\sin f dt dz, \quad (2.1)$$

where

$$h(r) = \arctan \sqrt{\left(1 + \frac{r^2}{a^2}\right)^{D-4} - 1}, \quad f = \frac{2\pi - \sqrt{2D-6} h(r)}{\sqrt{D-4}}. \quad (2.2)$$

Here  $d\Omega_{D-3}^2$  is a metric for the unit  $S^{D-3}$ , and  $\gamma$  and  $a$  are constants. It is clear that the case of  $D = 5$  is particularly simple, and it was previously obtained in [8] and further studied in [9, 10]. For simplicity, we shall focus our attention on  $D = 5$  and examine the properties that are important for our purpose.

We find that the metric for the Ricci-flat wormhole solution in five dimensions can be written as follows

$$ds^2 = dr^2 + (r^2 + a^2)d\Omega_2^2 - \frac{r^2 - 2asr - a^2}{r^2 + a^2}dt^2 - \frac{4acr}{r^2 + a^2}dt dz + \frac{r^2 + 2asr - a^2}{r^2 + a^2}dz^2, \quad (2.3)$$

where  $d\Omega_2^2 = d\theta^2 + \sin^2 \theta d\phi^2$  is the metric for a unit  $S^2$ , and  $c = \cosh \beta$ ,  $s = \sinh \beta$  and  $a$  are real constants, which, without loss of generality, are taken to be non-negative. Note we have made certain coordinate rotation and reparametrisation to obtain (2.3) from (2.1). The off-diagonal term in the metric implies that the solution is not static, but stationary.

There are two asymptotic regions, corresponding to  $r \rightarrow \pm\infty$ , in which the metric has the following form

$$ds^2 = -dt^2 + dz^2 + dr^2 + r^2 d\Omega_2^2. \quad (2.4)$$

This describes (Minkowskian) $_5$  if  $z$  is a real line, or (Minkowskian) $_4 \times S^1$  if  $z$  is a circle. We denote the  $r = \pm\infty$  worlds as  $W_+$  and  $W_-$ . There is no curvature singularity in the bulk between the two worlds. This can be seen from the Riemann square, given by

$$\text{Riem}^2 = \frac{24a^2(a^2 - 2r^2)}{(r^2 + a^2)^4}. \quad (2.5)$$

The two Riemann cubics can also be easily obtained, given by<sup>1</sup>

$$\begin{aligned}\text{Riem}_1^3 &\equiv R^\mu{}_\nu{}^\rho{}_\sigma R^\nu{}_\mu{}^\alpha{}_\beta R^\beta{}_\alpha{}^\sigma{}_\rho = \frac{192a^4r^2}{(r^2+a^2)^6}, \\ \text{Riem}_2^3 &\equiv R^\mu{}_\nu{}^\rho{}_\sigma R^\alpha{}_\mu{}^\beta{}_\rho R^\nu{}_\alpha{}^\sigma{}_\beta = -\frac{96a^4r^2}{(r^2+a^2)^6},\end{aligned}\quad (2.6)$$

The absence of the parameter  $\beta$  in the Riemann curvature implies that it is trivial. Indeed, we can make the following coordinate (un)boost,

$$t \rightarrow \cosh \frac{1}{2}\beta t + \sinh \frac{1}{2}\beta z, \quad z \rightarrow \sinh \frac{1}{2}\beta t + \cosh \frac{1}{2}\beta z, \quad (2.7)$$

the metric becomes  $\beta$  independent, given by

$$ds^2 = dr^2 + (r^2 + a^2) d\Omega_2^2 - \frac{r^2 - a^2}{r^2 + a^2} dt^2 - \frac{4ar}{r^2 + a^2} dt dz + \frac{r^2 - a^2}{r^2 + a^2} dz^2. \quad (2.8)$$

However, the off-diagonal term, which is crucial for the regularity of the solution, can not be removed by any real coordinate transformation. We can instead make a complex coordinate transformation

$$\begin{aligned}t &\longrightarrow \sqrt{\frac{1-is}{2}} x + \sqrt{\frac{1+is}{2}} z, \quad r \longrightarrow i\tilde{t}, \\ z &\longrightarrow \frac{s-i}{c} \sqrt{\frac{1-is}{2}} x + \frac{s+i}{c} \sqrt{\frac{1+is}{2}} z,\end{aligned}\quad (2.9)$$

the metric (2.3) then becomes diagonal, namely

$$ds^2 = -d\tilde{t}^2 + (a^2 - \tilde{t}^2) d\Omega_2^2 + \frac{a - \tilde{t}}{a + \tilde{t}} dx^2 + \frac{a + \tilde{t}}{a - \tilde{t}} dz^2. \quad (2.10)$$

This singular metric is a five-dimensional analogue of the Kasner universe in  $D = 4$ .

Although  $\beta$  is a trivial boosting parameter, we shall keep it in our discussion in this section, since it makes it easier to take the BPS limit and to obtain the supersymmetric purely-gravitational pp-wave.

The metric (2.3) has no degenerate surface associated with the radial coordinate  $r$ . The radius of the  $S^2$  never vanishes, with the minimum being  $a$ , measuring the size of the wormhole. The determinant of the metric in  $(t, z)$  directions is minus 1, *i.e.*

$$\det(g_{ij}) = -1, \quad \text{for } i, j = \{t, z\}. \quad (2.11)$$

The metric has no globally defined time coordinate. Whilst  $t$  is a good time coordinate for large absolute values of  $r$ , its rôle of time is relegated to  $z$  inside the wormhole. There is an ergo region

$$a(s - c) \leq r \leq a(s + c), \quad (2.12)$$

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<sup>1</sup>For the Schwarzschild-Tangherlini black hole, the ratio  $\text{Riem}^3/[\text{Riem}^2]^{3/2}$  is a pure numerical constant. Thus, our metric is not locally the black hole solution. We are grateful to Chris Pope for this observation.

in which  $g_{tt} \geq 0$ . In the region

$$-a(s+c) \leq r \leq a(c-s), \quad (2.13)$$

we have  $g_{zz} \leq 0$ . For the choice of  $z$  being circular, this implies a closed-timelike-circle (CTC); for it being a real line, the metric has no CTC. One might think that a linear combination of coordinates  $t$  and  $z$  could have a definite signature. This is not the case. Consider the Killing vector  $\ell = \mu \partial/\partial t + \nu \partial/\partial z$ . It is straightforward to obtain

$$r = \pm\infty : \quad \ell^2 = -\mu^2 + \nu^2, \quad r = 0 : \quad \ell^2 = \mu^2 - \nu^2. \quad (2.14)$$

It is perhaps instructive to use the light-cone coordinates

$$t - z = u, \quad t + z = v, \quad (2.15)$$

in which case, the metric becomes

$$ds^2 = \frac{a(c+s)r}{r^2+a^2} du^2 - \frac{r^2-a^2}{r^2+a^2} du dv - \frac{a(c-s)r}{r^2+a^2} dv^2 + dr^2 + (r^2+a^2) d\Omega_2^2. \quad (2.16)$$

The Killing vector  $\partial/\partial u$  is spacelike in the region  $r \in (0, +\infty)$ , timelike in  $(-\infty, 0)$  and null at  $r = 0$  and  $r = \pm\infty$ . The Killing vector  $\partial/\partial v$ , on the other hand, is timelike in  $(0, \infty)$ , spacelike in  $(-\infty, 0)$  and also null at  $r = 0$  and  $r = \pm\infty$ .

By studying the sub-leading structure of the metric at the infinities, we obtain the mass and linear momentum per unit  $z$  length, measured in either the  $W_+$  or the  $W_-$  world. They are given by

$$M_{\pm} = \frac{3}{32\pi} \int_{r \rightarrow \pm\infty} *dK_t = \pm \frac{3}{4} a s, \quad P_{\pm} = \frac{3}{32\pi} \int_{r \rightarrow \pm\infty} *dK_z = \pm \frac{3}{4} a c. \quad (2.17)$$

Here  $K_t$  and  $K_z$  are 1-forms associated with the Killing vectors  $\partial/\partial t$  and  $\partial/\partial z$  respectively. Thus the wormhole is created by an infinitely-stretched string (or a finite closed string) with momentum along the string.<sup>2</sup> The mass, or more precisely the tension of the string, measured in the  $r \rightarrow +\infty$  world  $W_+$  is positive, but it is negative in the  $r \rightarrow -\infty$  world  $W_-$ . The evasion of the positive-energy theorem in the  $W_-$  world may be related to the fact that it is asymptotic  $(\text{Minkowskian})_4 \times S^1$ . Even if the coordinate  $z$  is infinitely stretched, it is singled out from the rest spatial coordinates near the asymptotic infinities. This is analogous to the Atiyah-Hitchin metric whose Euclidean “mass” is negative. The metric is nevertheless smooth without violating the positive energy theorem due to the fact that its

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<sup>2</sup>It is perhaps more appropriate to call the solutions wormstring or wormtubes; however, we shall continue to use wormholes, for the disinclination of inventing new words.

asymptotic infinity is  $\mathbb{R}^3 \times S^1$ , rather than  $\mathbb{R}^4$ . It may also be related to the fact that we now have two asymptotic boundaries. It is instructive to note that we have  $M_+ + M_- = 0$ , which is consistent with that our solution is sourceless.

Another property of the wormhole is that

$$M^2 - P^2 = -\frac{9}{16}a^2 < 0. \quad (2.18)$$

This implies that the string is of tachyonic nature. Our wormhole solution may be viewed as a gravitational dual of a string tachyon state. In one limit, we can let  $\beta = 0$  and hence  $s = 0$  and  $c = 1$ , corresponding to a tensionless string with non-vanishing momentum. In the other limit, we can send  $\beta \rightarrow \infty$  and  $a \rightarrow 0$ , but keeping  $as \rightarrow q$  non-vanishing. In this limit, the BPS condition  $M = P$  is satisfied. The resulting metric following from (2.16) describes a supersymmetric pp-wave, given by

$$ds^2 = -dudv + \frac{2q}{r}du^2 + dr^2 + r^2d\Omega_2^2. \quad (2.19)$$

The two worlds  $W_+$  and  $W_-$  are disconnected in this limit at  $r = 0$ , which is a spacetime singularity.

We can perform the Kaluza-Klein reduction on the string direction  $z$ , and obtain an electrically-charged particle-like solution in four dimensions. The charge is larger than its mass, as in the case of all the charged particles observed in our universe. The solution is singular. It is rather common in string and supergravities that lower-dimensional singular solutions become regular after they are lifted up to higher dimensions [11].

Although we have concentrate the analysis on the  $D = 5$  solutions, The essential properties of the wormholes are shared by all higher dimensional solutions in (2.1). All these metrics for  $D \geq 5$  describe stationary inter-universe Lorentzian wormholes. Each of them have two asymptotic flat regions at  $r = \pm\infty$  and the bulk between these two regions has no singularity. There is no metric singularity associated with the radial coordinate  $r$ . The determinant of the metric in  $t$  and  $z$  directions is minus one. There is no globally defined time coordinate, and there exist BPS limits for the wormholes to become supersymmetric pp-waves for all dimensions.

To conclude this section, we note that the metric (2.3) can also be put in the following natural vielbein base

$$ds^2 = \frac{-[r(cdt - sdz) + adz]^2 + [r(cdz - sdt) - adt]^2}{r^2 + a^2} + dr^2 + (r^2 + a^2)d\Omega_2^2. \quad (2.20)$$

### 3 Geodesic motions and timelike trajectories

The geodesic motions for the  $D = 5$  wormhole solution were discussed in [9], and it was demonstrated that it is not geodesically traversable. In this section, we demonstrate that there exist accelerated time trajectories that cross over the wormhole and that the required total impulse is finite.

We begin by reviewing the geodesic motions, whose equations can be obtained from the Hamilton-Jacobi equations; they can also be equivalently derived from the Lagrangian

$$\begin{aligned} \mathcal{L} = \frac{1}{2}g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = & \frac{1}{2}\dot{r}^2 + \frac{1}{2}(r^2 + a^2)(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2) \\ & - \frac{r^2 - a^2}{2(r^2 + a^2)}\dot{t}^2 - \frac{2ar}{r^2 + a^2}\dot{t}\dot{z} + \frac{r^2 - a^2}{2(r^2 + a^2)}\dot{z}^2, \end{aligned} \quad (3.1)$$

where a dot denotes a derivative with respect to the geodesic parameter  $\lambda$ . Note that we set the trivial parameter  $\beta = 0$  in this section.

The equations of motion for  $\theta$  and  $\phi$  admit the following solution

$$\theta = \frac{1}{2}\pi, \quad \dot{\phi} = \frac{J}{r^2 + a^2}, \quad (3.2)$$

where  $J$  is an integration constant describing the orbiting angular momentum of the geodesic particle, the test particle travelling along the geodesics. The equations of motion for  $t$  and  $z$  imply that

$$\frac{r^2 - a^2}{r^2 + a^2}\dot{t} + \frac{2ar}{r^2 + a^2}\dot{z} = E, \quad \frac{r^2 - a^2}{r^2 + a^2}\dot{z} - \frac{2ar}{r^2 + a^2}\dot{t} = p, \quad (3.3)$$

where the integration constant  $E$  and  $p$  can be viewed as the energy and the linear momentum in the  $z$  direction of the geodesic particle. The Lagrangian itself is a constant for geodesic motions, *i.e.*,  $\mathcal{L} = -\frac{1}{2}\epsilon$ , where  $\epsilon = 1, 0, -1$  for the timelike, null and spacelike geodesics respectively. It follows that we have

$$\dot{r}^2 = -\epsilon - \frac{J^2}{r^2 + a^2} + V, \quad (3.4)$$

where the potential function  $V$  is given by

$$V = \frac{(E^2 - p^2)(r^2 - a^2) - 4aEr}{r^2 + a^2}. \quad (3.5)$$

For the particle to exist at asymptotic regions, we must have that  $E^2 - p^2 - \epsilon \geq 0$ . The potential  $V$  has a minimum at  $r = ap/E$ , with

$$V_{\min} = -E^2 - p^2. \quad (3.6)$$

Thus the wormhole is not traversable for both the timelike and null geodesics, corresponding to  $\epsilon = 1$  and  $0$  respectively. For the spacelike geodesics, corresponding to  $\epsilon = -1$ , the quantity  $\dot{r}$  can stay real for all real values of  $r$  provided that  $E^2 + p^2 \leq 1$ . Although we present here the explicit analysis for only the simplest case with  $\beta = 0$ , the general feature, namely that the wormholes are not traversable for the timelike and null geodesics, remains also true for all  $\beta \neq 0$ .

Having established the geodesic non-traversability, we now consider non-geodesic time-like trajectories.<sup>3</sup> For  $\beta = 0$ , the metric (2.16) can be written as

$$ds^2 = -\frac{(r du + a dv)(r dv - a du)}{r^2 + a^2} + dr^2 + (r^2 + a^2) d\Omega_2^2. \quad (3.7)$$

A timelike trajectory moving forward in time for large  $r$  then requires, for each positive  $d\tau$  where  $\tau$  is the proper time, that  $r du + a dv > 0$  and  $r dv - a du > 0$ . Such a trajectory does exist. For example, let us consider

$$u = \frac{\eta(r-a)^2}{a}, \quad v = \frac{\eta(r+a)^2}{a}, \quad (3.8)$$

where  $\eta$  is a positive constant. It follows that

$$r u' + a v' = r v' - a u' = \frac{2\eta(r^2 + a^2)}{a} > 0, \quad (r u' + a v')(r v' - a u') > (r^2 + a^2), \quad (3.9)$$

for all real  $r$  provided that

$$\eta > \frac{1}{2}. \quad (3.10)$$

This leads to a timelike trajectory going from  $r = +\infty$  to  $r = -\infty$ . To see this in more detail, let us consider the case with  $\theta = \frac{1}{2}\pi$  and  $\phi = 0$ . The trajectory motion is then governed by

$$-\frac{(r \dot{u} + a \dot{v})(r \dot{v} - a \dot{u})}{r^2 + a^2} + \dot{r}^2 = -1. \quad (3.11)$$

Thus we have

$$\dot{r} \sqrt{\frac{4\eta^2}{a^2} r^2 + (4\eta^2 - 1)} = -1. \quad (3.12)$$

It is clear that the velocity starts with zero at  $r = +\infty$  and reaches its maximum  $1/\sqrt{4\eta^2 - 1}$  at  $r = 0$  and reduces to zero again when  $r = -\infty$ . The acceleration is given by

$$\ddot{r} = -\frac{4\eta^2 a^2 r}{(4\eta^2 r^2 + a^2(4\eta^2 - 1))^2}. \quad (3.13)$$

Having obtained the comoving velocity and acceleration, it is straightforward to calculate the proper acceleration, given by

$$A^\mu = \dot{U}^\mu + \Gamma^\mu_{\nu\rho} U^\nu U^\rho, \quad (3.14)$$

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<sup>3</sup>We are grateful to Don Page for part of this discussion.



where  $U^\mu = \dot{x}^\mu$  for the coordinates  $x^\mu = (u, v, r, \theta, \phi)$ . We find that

$$A^2 \equiv g_{\mu\nu} A^\mu A^\nu = \frac{4\eta^2 a^2 \left( (4\eta^2 + 1)r^2 + a^2(4\eta^2 - 1) \right)}{(r^2 + a^2)(4\eta^2 r^2 + a^2(4\eta^2 - 1))^3}. \quad (3.15)$$

Thus the maximum acceleration occurs at  $r = 0$ , given by

$$A^2|_{\max} = \frac{4\eta^2}{a^2(4\eta^2 - 1)^2}. \quad (3.16)$$

The constraint  $\eta > \frac{1}{2}$  implies that for the timelike trajectories specified by the parameter  $\eta$ , the minimum value of maximum acceleration can be arbitrarily small, since  $\eta$  can be arbitrarily large.

We now examine the total proper impulse that is required for maintaining such an accelerated timelike trajectory. This is given by

$$\begin{aligned} \mathcal{I} = \int A d\tau &= \int_{-\infty}^{\infty} \frac{A}{\dot{r}} dr \\ &= \int_{-\infty}^{\infty} \frac{2a\eta\sqrt{(4\eta^2 + 1)r^2 + a^2(4\eta^2 - 1)}}{(4\eta^2 r^2 + a^2(4\eta^2 - 1))\sqrt{r^2 + a^2}} dr. \end{aligned} \quad (3.17)$$

This is finite because the integrand converges as  $1/r^2$  when  $r$  approaches  $\pm\infty$ . The minimum impulse that is required for these accelerated timelike trajectories is achieved when  $\eta \rightarrow \infty$ . In this case, we have

$$\mathcal{I}_{\min} = \int A d\tau = \int_{-\infty}^{\infty} \frac{a}{r^2 + a^2} dr = \pi. \quad (3.18)$$

It is worth noting, following from (3.17), that the total impulse required for crossing over the wormhole is independent on the size of the wormhole. This of course can be expected from the dimensional analysis.

## 4 Scalar wave propagation

Since particles under the pure gravitational influence without acceleration cannot pass the wormhole classically, we consider now the quantum tunnelling effects by studying the minimally-coupled scalar (of mass  $m$ ) wave equation, namely

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi) - m^2 \Phi = 0. \quad (4.1)$$

We consider a radial wave of frequency  $\omega$  and linear momentum  $p$ , *i.e.*,

$$\Phi = \frac{e^{i\omega t + ipz} P_\ell(\cos \theta)}{\sqrt{r^2 + a^2}} u(r) \quad (4.2)$$

The scalar wave equation is then reduced to

$$u'' - V_{\text{eff}} u = 0, \quad (4.3)$$

with the effective potential given by

$$V_{\text{eff}} = -(\omega^2 - p^2 - m^2) + \frac{a^2}{(r^2 + a^2)^2} + \frac{\ell(\ell + 1) + 2a^2(\omega^2 - p^2) - 4p\omega a r}{r^2 + a^2}. \quad (4.4)$$

At asymptotic regions, the effective potential becomes a negative constant  $-(\omega^2 - p^2 - m^2)$ . Defining  $k = \sqrt{\omega^2 - p^2 - m^2}$ , the wave solutions at  $r = \pm\infty$  have the following form

$$u_{\pm} = A_{\pm} e^{i k r} + B_{\pm} e^{-i k r}. \quad (4.5)$$

Thus the wave equation can be reduced to a one-dimensional quantum mechanical tunnelling system with a finite (non-singular) potential barrier. At  $r = 0$ , the effective potential is given by

$$V_{\text{eff}}(r = 0) = m^2 + \omega^2 - p^2 + \frac{\ell(\ell + 1) + 1}{a^2} > 0. \quad (4.6)$$

This is consistent with the earlier result that the wormhole is not traversable for geodesic motions.

To illustrate that the tunnelling takes place and obtain the transmission rate, let us consider a simpler case with  $p = 0$ . The effective potential resembles a square potential with height  $(\ell(\ell + 1) + 2a^2\omega^2)/a^2$  and characteristic width  $a$ . We can use the delta function approximation, and write

$$V_{\text{eff}} \sim -k^2 + S_V \delta(r), \quad \text{with,} \quad S_V \equiv \int_{-\infty}^{+\infty} (V_{\text{eff}} + k^2) dr = \frac{\pi}{2a} (1 + 2\ell(\ell + 1) + 4a^2\omega^2). \quad (4.7)$$

For such a potential, we may use the standard ansatz

$$u(r < 0) = c_T e^{i k r}, \quad \text{and} \quad u(r > 0) = e^{i k r} + c_R e^{-i k r} \quad (4.8)$$

to obtain

$$c_R = -\frac{1}{1 - i k / S_V}, \quad c_T = -\frac{i k / S_V}{1 - i k / S_V}. \quad (4.9)$$

Hence the reflection and the transmission rates are given by

$$R \sim 1 - \frac{k^2}{S_V^2}, \quad T \sim \frac{k^2}{S_V^2} = \frac{4(ka/\pi)^2}{(1 + 2\ell(\ell + 1) + 4a^2\omega^2)^2}. \quad (4.10)$$

Note that the delta function approximation is best justified when the radial wave length is much greater than the characteristic width of the potential barrier, *i.e.*  $ka \ll 1$ . The result shows that the transmission rate is smaller for the smaller wormholes. In the limit of  $a = 0$  when the wormhole disappears, the geodesic is complete at  $r = 0$ , and indeed the transmission rate vanishes.

## 5 Wormholes in five-dimensional supergravities

In this section, we consider wormhole solutions in five-dimensional  $U(1)^3$  supergravity. This theory, sometimes called the STU model, can be thought of as a truncation of maximal supergravity in five dimensions, in which two vector multiplets are coupled to the minimal supergravity. It is well known that the theory admits magnetic black string solutions. Our Ricci-flat wormhole can be viewed as a neutral tachyonic string. It is natural to charge it as in the case of black strings. We find that the wormhole solution with the magnetic charges is given by<sup>4</sup>

$$\begin{aligned} ds^2 &= -(H_1 H_2 H_3)^{-1/3} \left( \frac{r^2 - 2asr - a^2}{r^2 + a^2} dt^2 + \frac{4acr}{r^2 + a^2} dt dz - \frac{r^2 + 2asr - a^2}{r^2 + a^2} dz^2 \right) \\ &\quad + (H_1 H_2 H_3)^{2/3} \left( dr^2 + (r^2 + a^2) d\Omega_2^2 \right), \\ X_i &= H_i (H_1 H_2 H_3)^{-1/3}, \quad F_{(2)}^i = q_i \Omega_{(2)}, \quad H_i = \alpha_i - \frac{q_i}{a} \arctan\left(\frac{r}{a}\right). \end{aligned} \quad (5.1)$$

The solution is singular for any  $H_i = 0$ , which can be avoided provided that

$$\alpha_i \geq \frac{\pi |q_i|}{2a}. \quad (5.2)$$

Since the  $H_i$ 's approach constant when  $r \rightarrow \pm\infty$ , the solution again describes wormholes that connect two asymptotic flat spacetimes. The essential features of the Ricci-flat wormhole are retained, since the  $H_i$ 's are finite but non-vanishing. In the limit of  $a = 0$ , the solution becomes the supersymmetric black string whose near-horizon geometry is  $\text{AdS}_3 \times S^2$ . Indeed, it was shown in [11] that dilatonless magnetic BPS  $p$ -branes have no curvature singularity, and can be viewed as a wormhole, but with the inner and outer horizon worlds identified. In the BPS limit described in section 2, the solution becomes a magnetically-charged string with a pp-wave propagating along the string, with the near horizon geometry as a direct product of the extremal BTZ black hole (locally  $\text{AdS}_3$ ) and  $S^2$ .

A particular interesting case arises when the equality in (5.2) is reached for all  $i$ . In this case, in the asymptotic region  $r \rightarrow \infty$ , we have  $H_i \sim 1/r$ . It follows that the metric is asymptotically  $\text{AdS}_3 \times S^2$ . On the asymptotic region  $r \rightarrow -\infty$ , the  $H_i$ 's are constants, and hence the metric is flat. Thus the solution describes a wormhole that connects a  $\text{AdS}_3 \times S^2$  at one end to the flat Minkoskian spacetime in the other end.

Using the solution generating procedure that was developed in [13], we can also easily charge the solution with electric charge so that it becomes a solution of the  $D = 5$  minimal

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<sup>4</sup>We follow the convention of [12] with the cosmological constant turned off.

supergravity. Starting from the neutral wormhole (2.3), the resulting solution is given by

$$\begin{aligned}
ds^2 &= -\frac{f}{H^2} \left( dt + 2ac \left( \frac{\tilde{c}^3 r dz}{r^2 - a^2 - 2ars} + \tilde{s}^3 \cos \theta d\phi \right) \right)^2 \\
&\quad + H \left( dr^2 + (r^2 + a^2)(d\theta^2 + \sin^2 \theta d\phi^2) + \frac{1}{f} dz^2 \right), \\
A &= \frac{2\sqrt{3} a \tilde{c} \tilde{s}}{H} \left( \frac{a + rs}{r^2 + a^2} dt - \frac{rc \tilde{c}}{r^2 + a^2} dz - c \tilde{s} \cos \theta d\phi \right), \\
H &= 1 + \frac{2a(a + rs) \tilde{s}^2}{r^2 + a^2}, \quad f = \frac{r^2 - a^2 - 2ars}{r^2 + a^2}.
\end{aligned} \tag{5.3}$$

Here  $\tilde{c}$  and  $\tilde{s}$  are the boost parameters satisfying  $\tilde{c}^2 - \tilde{s}^2 = 1$ . The metric has a curvature power-law singularity when  $H = 0$ , but this can be avoided if the boosted parameter is less than a critical value, namely

$$\tilde{s}^2 < \frac{1 + c}{s^2}. \tag{5.4}$$

With this condition,  $H$  is non-vanishing, and the solution describes a charged wormhole. The mass, charge and momentum are given by

$$M_{\pm} = \pm \frac{3}{4} as(1 + 2\tilde{s}^2), \quad Q_e^{\pm} = \pm \frac{1}{2} \sqrt{3} as \tilde{c} \tilde{s}, \quad P_{\pm} = \pm \frac{3}{4} ac \tilde{c}^3. \tag{5.5}$$

In addition, there is a magnetic charge, given by

$$Q_m = \frac{1}{8\pi} \int_{r \rightarrow \infty} F_2 = \sqrt{3} ac \tilde{c} \tilde{s}^2. \tag{5.6}$$

The existence of this magnetic charge is responsible for that the metric is not asymptotically flat, but has instead two Gödel-like universes. In the appendix, we present the general wormhole solution in  $U(1)^3$  theory that carries both electric and magnetic charges.

## 6 Conclusions

In this paper we construct stationary Ricci-flat inter-universe Lorentzian wormholes in all  $D \geq 5$  dimensions. We focus our analysis on  $D = 5$ . The metric smoothly connects two asymptotic flat spacetimes. The solution can be viewed as supergravity dual of a string tachyon state whose linear momentum is larger than its tension. In the BPS limit, the solution becomes a supersymmetric pp-wave and the two worlds are then disconnected by the pp-wave singularity.

The wormholes are not traversable for the timelike and null geodesics. However, we show that there exist accelerated timelike trajectories that traverse from one asymptotic region to the other. The minimum value of the maximum acceleration of various trajectories can

be arbitrarily small. For the accelerated timelike trajectories that we obtained, the total impulse required for cross over the wormhole is finite and independent on the size of the wormhole, and its minimum value is  $\pi$ .

We further study the wave equation of the minimally-coupled massive scalars. We demonstrate that their quantum tunnelling between the two worlds must take place.

We also obtain charged wormhole solutions in five-dimensional supergravities. Although the metric becomes more complicated, the essential feature of the Ricci-flat wormholes is retained. With appropriate choice of parameters, we find that there exist wormholes that connect  $\text{AdS}_3 \times S^2$  in one asymptotic region to flat Minkoskian spacetime in the other. It is of interest to study the implication of such a geometry on the AdS/CFT correspondence.

Our results indicate that there exist a large class of smooth Lorentzian wormhole solutions in higher dimensions that are supported by less exotic energy-momentum tensors. If we reduce our wormhole solutions, with or without charges, on  $t$  and  $z$  directions, we obtain a Euclidean wormhole in three dimensions supported by a scalar coset. General Euclidean wormholes in Euclidean supergravities obtained from Kaluza-Klein reductions on tori that include the time direction [14] have been recently studied in [6, 7]. These solutions typically have singularities associated with the scalar coset, unless some liberal analytical continuation is performed which may be inconsistent with supersymmetry and U-dualities [6]. However, our results demonstrate that some of these lower-dimensional singular solutions can become smooth Lorentzian wormholes when they are lifted up to higher dimensions.

It is of great interest to classify these Lorentzian wormholes in supergravities in higher dimensions and investigate their properties such as the stability, geodesic traversability and the rôle they play in string theory.

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## Appendix

### A Wormholes with both electric and magnetic charges

Here we present the general wormhole solution in five-dimensional  $U(1)^3$  supergravity that carries both electric and magnetic charges. It is given by

$$\begin{aligned}
ds^2 &= (h_1 h_2 h_3)^{1/3} (H_1 H_2 H_3)^{2/3} \left[ dr^2 + (a^2 + r^2) d\Omega_2^2 + \frac{dz^2}{H_1 H_2 H_3 f} \right] \\
&\quad - \frac{f(dt + \omega)^2}{(h_1 h_2 h_3)^{2/3} (H_1 H_2 H_3)^{1/3}} ; \\
\omega &= c_1 c_2 c_3 \left\{ \left[ f_z - \frac{t_2 t_3}{H_1} - \frac{t_1 t_3}{H_2} - \frac{t_1 t_2}{H_3} \right] dz + (2act_1 t_2 t_3 - t_1 q_1 - t_2 q_2 - t_3 q_3) \cos \theta d\phi \right\} ; \\
A_i &= \frac{c_i s_i [1 - H_i^2 f_t(r)] dt}{h_i} + \frac{c_1 c_2 c_3 [t_j H_j + t_k H_k + (t_1 t_2 t_3 - t_i H_i f_z) f]}{h_i H_j H_k} dz \\
&\quad + \frac{c_1 c_2 c_3 [q_i - 2act_j t_k + t_i (t_j q_j + t_k q_k) H_i^2 f_t]}{h_i} \cos \theta d\phi \quad ; \quad j \neq k \neq i \\
X_i &= \frac{(h_1 h_2 h_3)^{1/3} H_i}{(H_1 H_2 H_3)^{1/3} h_i} \quad ; \quad H_i = a_i + \frac{q_i}{a} \tan^{-1} \left( \frac{r}{a} \right) \quad ; \quad h_i = c_i^2 - s_i^2 H_i^2 f_t ; \\
f_t &= \frac{f}{H_1 H_2 H_3} \quad ; \quad f_z = \frac{2acr}{r^2 - a^2 - 2ars} \quad ; \quad f = \frac{r^2 - a^2 - 2ars}{a^2 + r^2} \quad ; \quad t_i \equiv \frac{s_i}{c_i} . \quad (\text{A.1})
\end{aligned}$$

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